Binomial distribution (1)

Binomial distribution is also known as the Bernoulli distribution is a discrete probability distribution.

Binomial distribution is based on following condition.

(a) The random experiment is performed repeatedly a finite and fixed number of times

(b) each observation falls into one of two categories either called success or failure

(c) The probability of success in each trial remains constant.

(d) The trial are independent

Formula of Binomial distribution

$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

where n = number of trials

p = Probability of success

$q = 1-p$ = Probability of failure

x = discrete random variable

r = number of success in n trials

Formula of mean, standard deviation and skewness in the case of Binomial distribution,

$\mu = np$, variance $= npq$

standard deviation $= \sqrt{npq}$, skewness $= \frac{q-p}{\sqrt{npq}}$

Properties of Binomial distribution

1. For Binomial distribution, variance < mean
2. Binomial distribution is symmetrical $p=q=\frac{1}{2}$ +vely skewed for $p<\frac{1}{2}$ -vely skewed $p>\frac{1}{2}$

(3) If n is large and neither p nor q is too close to zero, the binomial distribution can be closely approximated by a normal distribution with a standardised variable

$$z = \frac{r - \mu}{\sigma}$$

$$= \frac{r - np}{\sqrt{npq}}$$

<u>Example</u>

Q1 → A coin is tossed 6 times, what is the probability of obtaining 4 or more heads? What is the probability of getting at least two heads.

<u>Solution</u> . If we toss a coin, then the probability of getting head (success) $= \frac{1}{2} = p$

Probability of not getting head (failure) $= \frac{1}{2} = q$

From question we have to find the probability of obtaining 4 heads or more heads.

~~P(r = 4 heads) =~~ $n = 6,\ p = \frac{1}{2},\ q = \frac{1}{2}$ $\quad P[X=r] = {}^nC_r\, p^r q^{n-r}$

$$P(r = 4\text{ heads}) = {}^6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{6-4}$$

$$= {}^6C_4 \times \frac{1}{16} \times \frac{1}{4}$$

$$= \frac{\lfloor 6}{\lfloor 4 \lfloor 2} \times \frac{1}{64}$$

$$= \frac{6 \times 5 \cancel{\lfloor 4}}{\cancel{\lfloor 4} \times 2 \times 1} \times \frac{1}{64}$$

$$= \frac{30}{128}$$

$$= \frac{15}{64}$$

$$P(r=5) = {}^6C_5\left(\frac{1}{2}\right)^5\cdot\left(\frac{1}{2}\right)^{6-5}$$

$$= {}^6C_5\,\frac{1}{32}\times\frac{1}{2}$$

$$= \frac{6!}{5!\,1!}\times\frac{1}{64}$$

$$= \frac{6\,\cancel{5!}}{\cancel{5!}}\times\frac{1}{64}$$

$$= \frac{3}{32}$$

$$P(r=6) = {}^6C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{6-6}$$

$$= \frac{\cancel{6!}}{\cancel{6!}\,0!}\times\frac{1}{64}$$

$$= \frac{1}{64}$$

Hence probability of getting 4 or more heads

$$= P(r=4)+P(r=5)+P(r=6)$$

$$= \frac{15}{64}+\frac{3}{32}+\frac{1}{64}$$

$$= \frac{15+6+1}{64}$$

$$= \frac{22}{64} = \frac{11}{32}$$

$$= 0.343$$

(ii) Probability of getting of 2 heads

$$= 1-\{P(0\text{ head})+P(1\text{ head})\}$$

$$= 1-\left[{}^6C_0\left(\frac{1}{2}\right)^0\times\left(\frac{1}{2}\right)^{6-0} + {}^6C_1\times\frac{1}{2}\times\left(\frac{1}{2}\right)^{6-1}\right]$$

$$= 1-\left[{}^6C_6\times\left(\frac{1}{2}\right)^6 + {}^6C_1\times\left(\frac{1}{2}\right)^6\right]$$

$$= 1 - \left[\frac{1}{64} + \frac{6}{64}\right]$$

$$= 1 - \frac{7}{64}$$

$$= \frac{57}{64}$$

$$= 0.890$$

Q → 8 coins are tossed 256 times. Find the expected frequencies of success (getting a head) and tabulate the result obtained. Find the mean and standard ~~s~~ deviation of the fitted distribution.

Solution, $P[\text{success}] = \frac{1}{2}$, $q[\text{failure}] = \frac{1}{2}$, $N = 256$

$n = 8$

~~$P[r=8] =$~~

$$P[r] = {}^{8}C_{r}\, p^{r} q^{n-r}$$

$$= {}^{8}C_{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{8-r}$$

$$= {}^{8}C_{r}\left(\frac{1}{2}\right)^{r}\left(\frac{1}{2}\right)^{8-r}$$

$$= {}^{8}C_{r}\left(\frac{1}{2}\right)^{8}$$

$$= \frac{1}{256} \times {}^{8}C_{r}$$

Hence in 256 throws $f(r) = N \times \frac{1}{256}\, {}^{8}C_{r}$

Success.

$$= 256 \times \frac{1}{256}\, {}^{8}C_{r}$$

$$= {}^{8}C_{r}$$

Success	0	1	2	3	4	5	6	7	8
Expected frequency	${}^{8}C_{0}$	${}^{8}C_{1}$	${}^{8}C_{2}$	${}^{8}C_{3}$	${}^{8}C_{4}$	${}^{8}C_{5}$	${}^{8}C_{6}$	${}^{8}C_{7}$	${}^{8}C_{8}$

Mean, $\mu = np$

$$= 8 \times \frac{1}{2} = 4$$

$$S.D = \sqrt{npq} = \sqrt{8 \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{2} = 1.414$$